Logarithmic Modeling (2.14) + FRQ 2 + Calculator Practice Homework

You may use a calculator/Desmos throughout this entire assignment, though #1-6 do not need it.

In questions #1-4, match the numbered description on the left with a corresponding function on the right.

1. Each time the input value doubles, the output value increases by 1.
2. Each time the input value increases by 1, the output value doubles.
3. Each time the input value increases by 1, the output value increases by 2.
4. Each time the input value increases by 1, the rate of change increases by 2.

$f(x) = x^2 + 10$

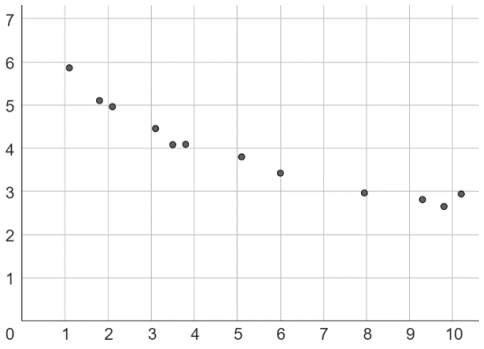
$g(x) = 3(2)^x$

$h(x) = \log_2(x) + 3$

$k(x) = 2x - 5$

5. The graph of $y = g(x)$ intersects the x-axis at the point $(9,0)$. For each time the input value of g is tripled, the output of g increases by 1. Which of the following could represent $g(x)$?
   (A) $g(x) = \log_3(x - 8)$
   (B) $g(x) = 3\log_9(x) - 3$
   (C) $g(x) = \log_3\left(\frac{x}{9}\right)$
   (D) $g(x) = \log_9\left(\frac{x}{3}\right)$

6. Consider the points shown to the right. The distribution of these points can be modeled well by a function of the form $f(x) = a - \log_b x$. Select the most reasonable choice of b.
   (A) 2
   (B) 3
   (C) 4
   (D) 5



7. The half-life of carbon-14 is k years. This means that given a sample, the mass of carbon-14 in the sample decreases in such as way that every k years the mass is halved. If a sample begins with 500 g of carbon-14, then the mass of carbon-14 after t years can be modeled by $C(t) = 500\left(\frac{1}{2}\right)^{\frac{t}{k}}$.
   a. Given that there will be 308.2 g of carbon-14 remaining in the sample after 4000 years, find the value of k.
   b. Given that there will be 308.2 g of carbon-14 remaining in the sample after 4000 years, find how long it will take for there to be only 200 g of carbon remaining in the sample.

8. Bernie deposits $\$1000$ in a savings account with a 4% APY interest rate. This means that the amount of money in the account increases by 4% every year.
   a. Write a function that models the amount of money in Bernie's account after t years.
   b. Write a function that models the amount of money in Bernie's account after t months.
   c. How much money is in Bernie's account after 30 months?
   d. How long will it take for Bernie's initial deposit of $\$1000$ to double in value? Give your answer in years.

9. In 2007, a fruity computer company began selling a new type of cell phone. To reach one million global sales, it took 7 months. To reach five million sales, it took 17.5 months. Below is a table showing how long it took to reach certain "global sales" milestones in terms of months since the product launch.

Global Sales (millions)	1	5	10	20	50	100	200
Time (months)	7	17.5	23.5	29.5	38	44.75	51.25

   Use a graphing utility to construct a logarithmic function of the form $T(n) = a + b\ln(n)$ that models the amount of time T (in months) it took to reach n global sales (in millions of phones).

   a. Use the model to predict how many months it took to reach 300 million sales.
   b. Use the model to predict how many sales had been made after 55 months. Round to the nearest million.
   c. Find the average rate of change of the model $T(n)$ over the interval $15 \leq n \leq 25$. Include units.
   d. The function $T(n)$ has a vertical asymptote at $n = 0$. Does that make sense in the context of this question?

10. The strength of earthquakes is described using the "Richter Scale." Larger numbers on the Richter scale reflect an earthquake that was "stronger" and therefore released more energy.
    - The 1989 California earthquake registered 6.9 on the Richer scale and is estimated to have released 562.34 terajoules of energy.
    - The 2020 Salt Lake City earthquake registered 5.7 on the Richter scale and is estimated to have released 8.91 terajoules of energy.
    - The 2024 earthquake off the east coast of central Florida registered 4.0 on the Richter scale and is estimated to have released 0.03 terajoules of energy.

    The amount of energy released by an earthquake can be converted to a Richter Scale description using a function of the form $R(E) = a + b\ln(E)$ where E is the amount of energy released and R is the Richter scale description for the given quake. Using a graphing utility and the three earthquakes given above, construct a model for this relationship.

    a. If an earthquake releases 1 terajoule of energy, predict what it registers on the Richter scale.
    b. If an earthquake registers as 6.0 on the Richter scale, predict how much energy it releases.
    c. How many times more energy does a 7.0 Richter scale earthquake release than a 6.0 earthquake?
    d. How many times more energy does a 6.0 Richter scale earthquake release then a 5.0 earthquake?

11. Mavis's New Year's resolution is to become a faster typist. Every day, she practices and then takes a test to measure her typing speed. After 5 days of practice ($t = 5$), she can type at a speed of 46 WPM. After twelve days of practice ($t = 12$), she can type at a speed of 54 WPM.

    The speed at which Mavis can type can be modeled by $S(t) = a + b\log_2(t+1)$, where $S(t)$ is her speed, in WPM, and t is days since she began becan practicing.

    (A) (i) Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $S(t)$.
    (ii) Find the values for a and b as decimal approximations.

    (B) (i) Use the given data to find the average rate of change of Mavis's typing speed, in WPM per day, from $t = 5$ to $t = 12$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
    (ii) Use the average rate of change found in part B(i) to estimate Mavis's typing speed, in WPM, for $t = 14.5$ days. Show the work that leads to your answer.
    (iii) Compare the estimate found in (ii) to the value given by the model, $S(14.5)$. Using characteristics of the average rate of change and characteristics of the logarithmic model, explain why the estimate based on average rate of change overapproximates the value of $S(14.5)$. Your explanation should include a reference to the graph of $S(t)$.

    (C) Mavis plans to stop her daily typing practices once her typing speed reaches 80 WPM. Explain how this information may be used to determine a boundary for the domain of S.

Logarithmic Modeling HW Answers

1. $h(x)$
2. $g(x)$
3. $k(x)$
4. $f(x)$
5. C
6. A
7.
   a. $k = 5730.15634$ (you can round to 5730.156 but use more than three decimal places when you use this number for part b. Ideally, use the v function in your calculator)
   b. 7574.855 years
8.
   a. $A(t) = 1000(1.04)^t$
   b. $A(m) = 1000(1.04)^{\frac{m}{12}}$
   c. $1000(1.04)^{2.5} = 1103.020$
   d. 17.673 years
9. (note: this question is based on data from the sale of iPhones, but numbers have been messed with a little. Thus why it says "a fruity company" rather than Apple. But the numbers are almost right for the iPhone)
   a. 53.524 months
   b. 357 million
   c. $\frac{T(25)-T(15)}{25-15} = 0.434 \frac{\text{months}}{\text{million sales}}$
   d. No, the asymptote does not make sense in the context of this question. It would suggest that the time at which there were close to nearly zero sales (n approaching 0) corresponds with a massively negative number of months before launching ($T(n)$ approaching $-\infty$). Anywhere the model has a negative output, the model is not appropriate to use to describe the sales of this phone.
10.
    a. 5.040
    b. 25.865 TJ
    c. 7.0 earthquake releases 767.320 TJ while 6.0 earthquake releases 25.865 TJ.
    $$\frac{767.320}{25.865} = 29.666$$
    The 7.0 earthquake releases approximately 29.666 times more energy than the 6.0 earthquake.
    d. 6.0 earthquake releases 25.865 TJ while 5.0 earthquake releases 0.872 TJ
    $$\frac{25.865}{0.872} = 29.666$$
    The 6.0 earthquake releases approximately 29.666 times more energy than the 5.0 earthquake.

11.

A.

i. $46 = a + b\log_2(5+1)$
$54 = a + b\log_2(12+1)$

ii. $a = 27.461$ and $b = 7.172$

B.

i. $\frac{S(12)-S(5)}{12-5} = \frac{54-46}{12-5} = \frac{8}{7} = 1.143$

ii. $\frac{S(14.5)-46}{14.5-5} \approx \frac{8}{7}$
$S(14.5) - 46 \approx 10.857$
$S(14.5) \approx 56.857$

iii. S is a concave down function. The estimates based on average rate of change are the outputs of the secant line passing through S at $t = 5$ and $t = 12$. Since S is concave down, the secant line is above the graph of S for $t > 12$. Thus, at $t = 14.5$ the secant line overapproximates the value of $S(14.5)$.

C. Setting $S(t) = 80$ yields a solution of $t = 159.423$. Mavis will stop practicing when $t = 159.423$. For $t > 159.423$, the model increases. But in reality, Mavis will have stopped practicing and therefore her typing speed will not continue increasing beyond 80 WPM. So, we should restrict the domain of S to $t \leq 159.423$.